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How To Diagnose The Thinking Of Cardinal Number To Children Of 3, 4 And 5 Years?

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Abstract: *A battery of tasks is presented to diagnose the logical thinking of the cardinal number in students aged 3 to 7 years. We intend to make known some scales of measurement that diagnose the students according to different ages. The diagnosis in schoolchildren aged 3 to 7 years will be made according to periods of six months and attending to a type task that entails the mathematical logical scheme of cardinal number. For each age period there is a task, in all of them, given a set, the school must answer the question "how many are there?", And reciprocally, given a number the child must determine a set whose cardinal is the given number. In the periods of age of 3 and 4 years it is considered an evolved counting strategy. In the periods corresponding to 3 years, the numbers that appear in the tasks do not exceed 5, in the first period of 4 years do not exceed 10, in the second they reach 20, they are numbers corresponding to the second decade. For 5 and 6 years the most evolved strategy is the progressive count. In the first 5-year period the numbers do not go from 10, in the second period up to 20. For 6 years, first with numbers up to 50 and then any number up to 100. For each age period, the corresponding task consists of an initial situation that the child must solve. Through the study of the strategy used in its resolution will be made the diagnosis. If the strategy is evolved the task corresponding to the next period of age is passed to evaluate the real age in relation to the cardinal knowledge of the school. If the student is not able to solve this initial situation, an analysis of the errors is made and through that the appropriate previous period is decided, presenting the task corresponding to that period, then, if the child is able of correctly realizing the initial situation of the task corresponding to that period, it analyzes the strategy that has followed for the resolution and through it the diagnosis is made.*

Keywords: *cardinal number, diagnosis, logical mathematical thinking, Early Childhood Education*

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I. INTRODUCTION

The logical mathematical thinking of the cardinal number is linked to the mathematical conception of the natural number as the property that all equipotent sets have in common. All these equipotent sets together form an equivalence class and the membership property of a set to the class is its cardinal number. Therefore, the cardinal number is always linked to a set, just as any set always has associated a number that is its cardinal, that is, the number of elements that the set has, in this sense the cardinal number gives response to the question how many there are? or how many elements has the set, and vice versa given a number can be determined a set with that number of elements (Clark y Grossman, 2007; Escalona, 2016; Feigenson y Carey, 2005).

In order to determine the appropriate tasks to carry out the diagnosis of the logical mathematical thought of the cardinal number, we have based ourselves on theories and models on the development of the counting action in the students, since the counting is a valid instrument to determine the cardinal of a set and vice versa given a number, through the count, it is possible to determine a set with that cardinal (Escalona y Fernández, 2017).

In the study of the development of the cardinal number in the child, there have been lines of investigation based on the action of counting, which have been projected in the works on teaching and learning of this concept and also on its diagnosis. All of these researches are within the model of skills integration widely followed today (Cordes y Gelman, 2005; Escalona, 2015; Gelman y Gallistel, 2004; Le Corre y Carey, 2007).

In this model part of the count is taken as a primary conception in the development of the number (taking into account that this ability tends to appear early in childhood development), from which one reaches the understanding of its meaning as a quantifying operator, this meaning of quantification identifies the number with the cardinal number and thus the purpose of the count is to calculate the cardinal of a set (Clark y Grossman, 2007; Feigenson y Carey, 2005; Fuson, 1988), that is, this theoretical reference would lead to the construction of models of number development starting from the counting action and using the counting itself as a "quantifying operator" (Clark y Grossman, 2007).

According to these investigations, the activity in the Children's classroom requires a special attention to the action of counting and it means taking it into account in the way that the child travels towards the interiorization of the number; in this sense we are going to deepen how the counting is carried out in the child and how he conducts his numerical thinking.

If we take into account the development of the counting action in schoolchildren we can determine the task of diagnosis of the logical mathematical thinking of the corresponding cardinal number for a given age. Gelman and Gallistel's counting principles and Fuson's numerical sequence domain levels are part of the investigations in the information processing line that determine the counting models linked to the cardinization of sets (Fuson, 1988; Gelman y Gallistel, 2004; Le Corre y Carey, 2007). We will use these theories to determine the situations included in each of the tasks corresponding to each age for diagnosis. The development in the child of the action of counting is marking the guidelines to be followed in the proposal of activities, for example, according to these theories, a boy or girl of 3 years dominates the sequence in the section 1-5 therefore the situations in the diagnostic tasks of this age try to determine sets with a given cardinal between 1 and 5 elements and determine how many elements has a set with less than 5 objects.

Gelman and Gallistel (2004) defend the existence of principles in the count and the role that plays in the determination of the characteristics that must have a correct execution of the counting action to arrive at the cardinization of the sets. We will base ourselves on these principles to determine the appropriate tasks for the diagnosis of the logical mathematical thinking of the cardinal number in schoolchildren.

We will focus on the action of counting and the mental schemes that this entails. To count is to assign a term of the numerical sequence to an object other than a well-defined set. Therefore, in order to count we must first have a succession of numerical terms and a well-defined set of objects whose elements are to be put into correspondence one by one with the numerical sequence.

The sequence in question is the conventional numerical sequence "one, two, three, four, five, ...". This sequence consists of: the terms and order of the recitation of these terms. For the assimilation of this sequence we have a beginning of the count and it is the principle of stable order that tells us that the words used to count must be produced with an order established between term and term.

Children gradually acquire this sequence; so around three and a half years dominate a first conventional and stable section, from one to five, these children continue the sequence with a second unconventional but stable section and end with a third non-conventional and non-stable (Gallistel and Gelman, 2005). Here the first indicative appears in the diagnosis tasks, for the schoolchildren from 3.6 years old to 4.0 the sets can not pass from 5 elements because their stable and conventional stage reaches up to 5. Previous to this is the section 1-3 that is left for children from 3 years to 3 ½ years.

From now on we will try to relate the numerical sequence to a set of objects. It is not just a recitation of numbers, but a term of the sequence must be associated with each of the objects; therefore, it is necessary to establish a bijective application between the set of objects and part of the numerical sequence. For this we have another principle of the count, namely: the one-to-one correspondence principle that tells us that each object in the set to be counted must receive one and only one term of the numerical sequence (Cordes and Gelman, 2005). Regarding the evolutionary thinking of the child, this principle is related to the stable order, so if children dominate the sequence up to 5 then they are in a position to match each object in the set a number and if not can be assessed in the diagnosis through errors made in the performance of the task (Escalona, 2017).

The children, when beginning in this principle, normally touch the objects while they count them, for that reason a correspondence appears between the signaling of the object and the numerical term that is pronounced at the same time that is indicated and this is the labeling; so that one-to-one correspondence declines in a spatio-temporal correspondence. It is easier to do one-to-one correspondence when the objects are in the form of a row and this is included in the diagnosis tasks when the child makes mistakes with this principle and is the reason why it does not solve the first situation that serves to diagnose that it is in the level, being then when the situation of the set in row is presented (Hartmann, 2015; Patro and Haman, 2012).

In the act of counting intervenes, as we have just seen, a set of well-defined objects that will be counted. Therefore, we have another principle that will deal with the nature of these sets. This principle is the principle of abstraction that tells us that any collection of objects is an accounting set, the only thing required of the set is that it has its elements well differentiated from each other. For example, we can count collections of objects, living beings, people, animals, plants, events that happened in time, sounds, etc. From the evolutionary point of view, schoolchildren succeed in counting when the sets have elements that they can see and touch, so in the initial tasks of diagnosis the sets present this characteristic so that later when the students are advancing in some sets presented in the tasks are replaced by numerical data and can no longer see and count their elements (Hartmann, 2015).

The culmination of the counting action is not given without an important fact and is that the count is an algorithm that facilitates us to calculate the cardinal of a set. It is here that the principle of cardinality intervenes that says that the last term of the count indicates the cardinal of the set. But there is more, no matter the order in which the count has been made; we always get the same number of elements whether we start with an object or another from the collection and this is collected in the principle of irrelevant order (Feigenson and Carey, 2005).

While the principles of stable order and one-to-one correspondence refer to "how to count", the principle of cardinality is related to an explicit purpose of the count: "to find out the number of elements in a collection".

When the child has a numerical sequence (principle of stable order), applies that sequence to a collection of objects (one-to-one correspondence principle) and counts any set of increasingly varied nature (principle of abstraction), will be in order to give the last word of said count a special meaning.

Therefore, these three principles prove to be necessary to understand the principle of cardinality but are not sufficient. And it is not enough since there are schoolchildren who, after having correctly counted a collection of objects and before the question: "How many are there?", Instead of saying the last word of the count, begin to count again In the tasks of diagnosis so that children apply the principle of cardinality and avoid re-counting elements of the collection, in these tasks in addition to cardinal a set is asked that given a number is obtained the cardinal of the set, for example "give me 3 letters of that pile".

This principle is achieved around 3 and a half years, giving three stages in its development (Le Corre and Carey, 2007):

I. Step of the action of counting to the cardination. At this stage the child considers that the last term used in the count is adequate to obtain the cardinal of the set. This stage is taken into account in the diagnosis when in the tasks corresponding to the first ages the child is only asked to cardine a set by counting.

II. Step from carding to counting. At this level there is a greater state of comprehension in the converse conversion of what was done in the first stage, that is, one moves from the cardinal to the meaning of that term as a result of counting. This occurs mainly with small collections, the child sees a set of four objects, for example, and says that there are four (the cardinal), then counts them and observes that the result of that count coincides with the cardinal of the set. Taking into account this stage in the diagnosis tasks, the child is asked to take a set of 3 elements and then to say how many elements that collection has given.

III. The integration of both meanings. This step refers to the cumulative aspect of the count. Each term obtained by counting simultaneously carries a sense of carding. For example, if we have to count seven objects, when we count "one, two, ..., seven" we say there are seven because this is the last word of the count, but when counting, when we name the term "five" we have already counted five elements and that at that moment of the count there are five elements, and so with all terms up to seven. This cumulative aspect is evident in the following scheme (Figure 1):

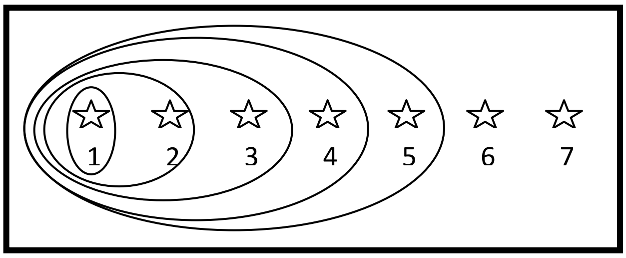

Figure 1. Cumulative sense of the count.

This cumulative sense is presented in the tasks of diagnosis linked to progressive counting, it is a matter of determining the quantity of a set knowing that a subset has a given quantity of elements given as data and from that number must determine the cardinal of the set. These tasks are presented to schoolchildren from 5 years of age, coinciding with the most advanced age of the children diagnosed with the last stage of development of the cardinality principle.

In the diagnosis the schoolchildren must determine the cardinal of the set through counting and build sets given their cardinal.

Closely linked to the principles of counting we have the levels of domain of the numerical sequence given by Fuson (1988). It is linked to the principle of stable order and is necessary to perform the counting or counting action to determine the cardinal of the set. The period of elaboration of the numerical sequence, according to Fuson, Richards and Briars (1982), is subdivided into five levels:

1. String level, in which the numbers are not subject to reflection and can only be issued neatly.

At this level you can only issue the sequence as an "all" without differentiating the numeric words that appear within it. The lack of differentiation makes the terms considered as labels without any comparative link between them.

This leads to the "non-obtaining" of success in tasks related to the counting of the lack of coordination of the two basic components of the count: one-to-one correspondence and sequence of numerals.

2. Unbroken chain level, during which the numerals become objects of reflection, since the process of differentiation between the terms of the sequence has begun.

Each of the words that are emitted within the sequence are terms distinguishable from each other, and thus the sequence is not constituted as a "whole" but is composed of a succession of terms.

Such differentiation of terms allows, among other things, that one can establish a one-to-one correspondence between the terms of the sequence and the objects of an accounting collection.

3. Breakable chain level, at which point the parts of the sequence can be issued starting from any point in the sequence of numerals, instead of having to always start with the first element as it did in the previous level.

There is a greater understanding of the relationships between numerical words within the sequence.

4. Numberable chain level, level at which numerals reach a higher degree of abstraction and become units that can be counted.

It can be counted from any term "a" until you reach another term "b". When you have to continually remember the term of arrival, new connections appear between a certain term, the previous to this and the next. If you have to reach the "b" term, when counting and reaching "b-1" you must know that the next of that number is "b". But the opposite relationship, that is, a child who has the ability to count from a term "a" n-terms and give another term "b" in response, knows that the term "b-1" is prior to "b" and that when it arrives to reach that term, the following will be with which it is to finalize.

5. Bidirectional chain level, which is the culmination of the processing process, since numerals can be issued with great ease and flexibility in any direction (increasing or decreasing).

At this level the culmination of the phase of elaboration of the sequence occurs, each term in the sequence occupies a certain place because it is posterior to all that precede them and previous to all that happen to them.

In the proposed diagnostic tasks, the domain sequence levels are taken into account, especially when evaluating the type of strategy used, such as the progressive count, for example, in the situation "Here there are 5 more until you reach to 10 ", to solve this situation. A boy or girl who correctly solves this situation is in the numberable chain level in the 1-10 section since he must count from 5 and stop at 10 (Escalona, 2015).

TASKS FOR DIAGNOSIS

The diagnosis in schoolchildren aged 3 to 7 years will be made according to periods of six months. Given any student of a certain age, we will say that he has acquired cardinal numerical competence if he has successfully passed the "type task" proposed in the diagnosis with the mathematical logical scheme of the cardinal number.

At each age a type task corresponds. This task consists of the following: a situation 1 that if done correctly by the school will indicate that the diagnosis is positive and to see the level that is analyzed the strategy followed for resolution (poorly developed, typical of age and more evolve) and that the school has a suitable level of cardinal knowledge according to the strategy followed.

If the student does not correctly do the situation 1, he / she moves to situation 2 which also carries the cardinal number but is easier than the situation 1. If he / she does not correct the situation 2, the diagnosis is that the student is not at the corresponding level and then the errors are analyzed. If, on the other hand, you correctly do situation 2, you move to situation 3.

Situation 3 carries the same mathematical logical scheme as situation 1 but it is easier than this although more difficult than situation 2. If the child does not perform the situation correctly 3 the diagnosis is negative and the errors are analyzed. If, on the other hand, it is performed correctly, then situation 1.

If after all this process the school does not correctly perform the situation 1 the diagnosis will be negative whereas if it is performed correctly then the diagnosis will be positive and the strategy followed.

Then, by means of figures, we present in a schematic way the diagnostic tasks corresponding to each age and we put cases of schoolchildren to whom the diagnosis has been made. The material used in the diagnosis are cards in which there are drawn a red or green apple.

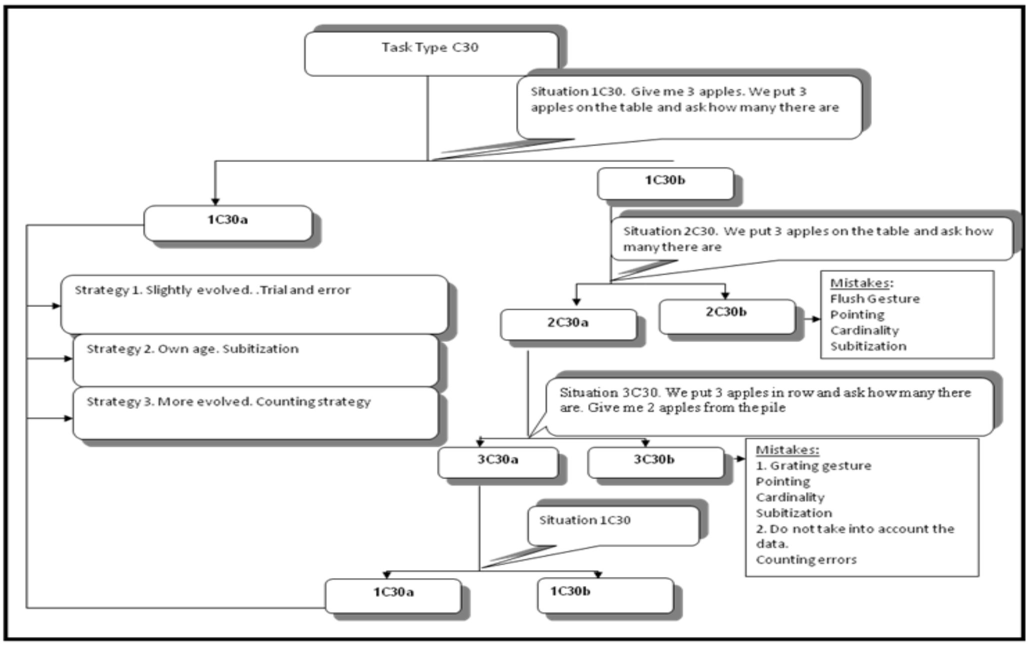

Figure 2. Task to diagnose the cardinal in schoolchildren from 3 years to 3 ½ years


Figure 3. Girl correctly counting 3 cards arranged in any way on the table

The case of Ma (3,2) a girl of 3 years and 2 months: the girl is able to take 3 letters from a pile "take a letter and say one, take another and say two and take another and say three", in addition it is able to count them correctly when they are disposed of any form on the table and to answer the question how many there ?. This girl has correctly performed situation 1 with the most evolved strategy that is counting. Therefore, his diagnosis is positive and he is more evolved in his cardinal thought than his age.

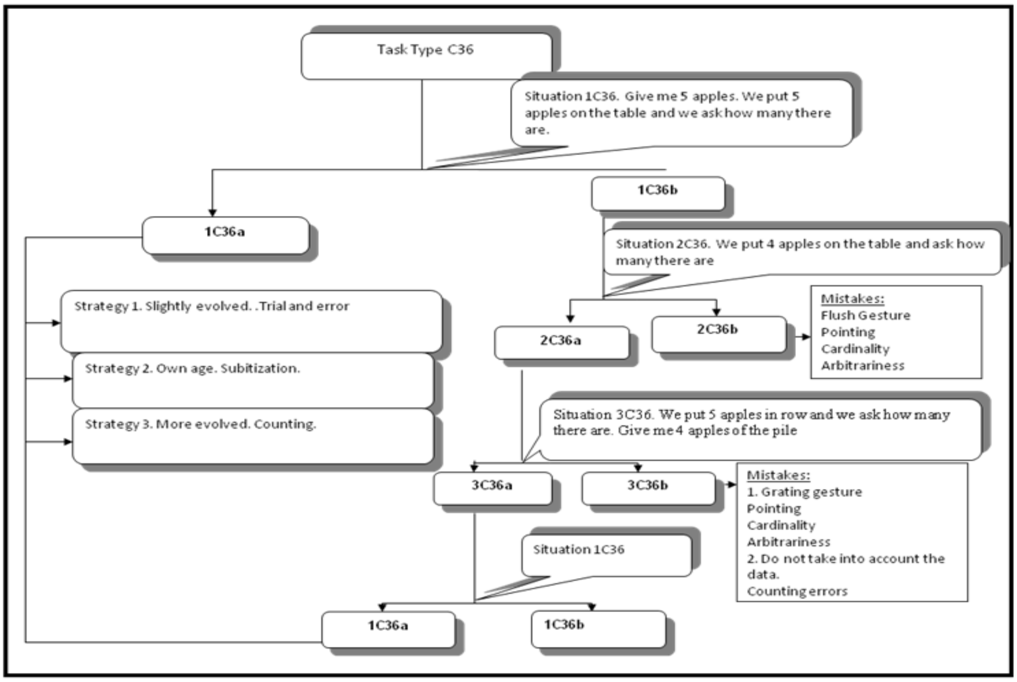

Figure 4. Task to diagnose the cardinal in schoolchildren aged 3 years and a half to 4 years

Figure 5. Girl counting cards in row and committing labeling error

Lau (3,7) is a girl of 3 years and 7 months who does not pass the diagnosis corresponding to her age because she is not able to correctly count 5 cards placed in a row because she labels with the same number two letters says "u-no "At the same time it points out letters one and two, so it is an error as to correspondence labeling / signage according to the principle of one-to-one correspondence of counting principles. The girl does not correctly perform labeling because, while pointing out two different tastings, the first and the second, she is using a single label "one".

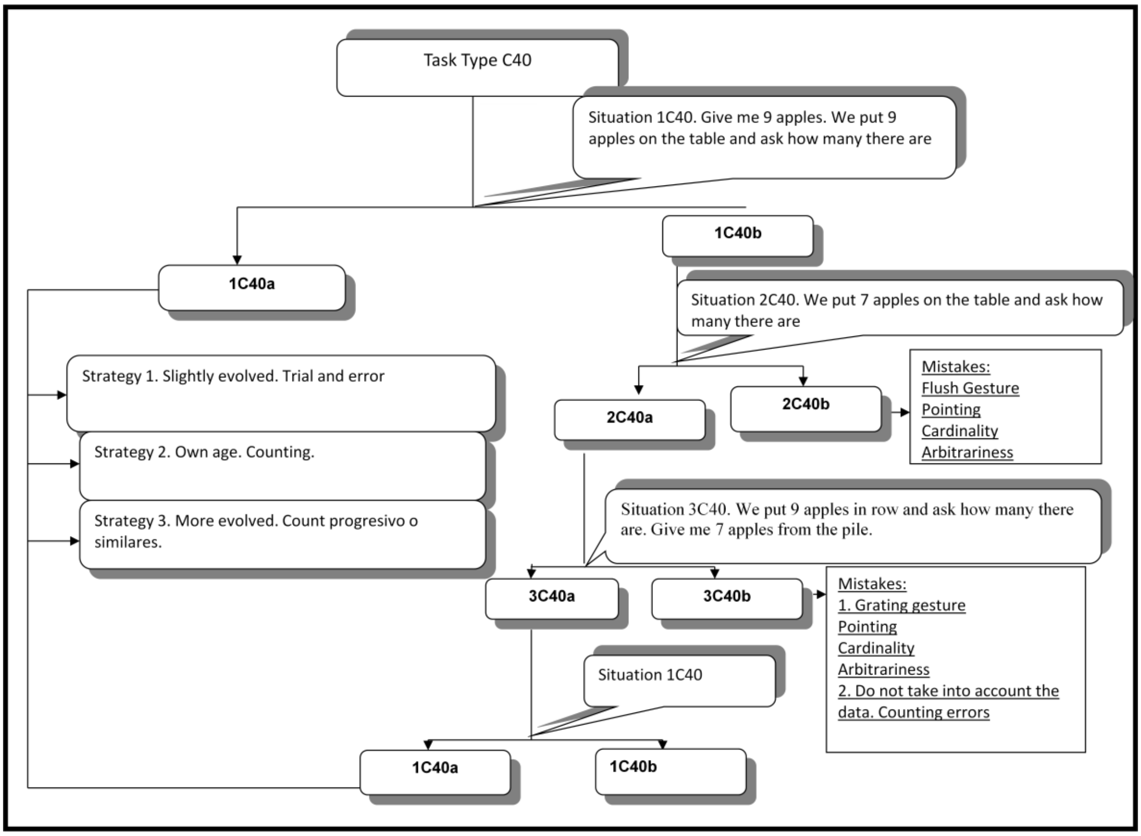

Figure 6. Task to diagnose the cardinal in schoolchildren aged 4 years to 4 ½ years

Figure 7. Error when not taking into account the data

Mig. (4.1) is a child who does not pass the diagnostic test because he does not take the data into account. When we say give me 9 letters the child picks a letter and says 1, picks another and says 2 and so on without stopping at 9, so that counting until reaching 15 is when the experimenter stops and tells you again it tells you. The test is repeated and the same procedure is repeated. This child is not in the numberable chain level in the 1-10 section of the Fuson number sequence domain levels, since he is not able to stop at a given number.

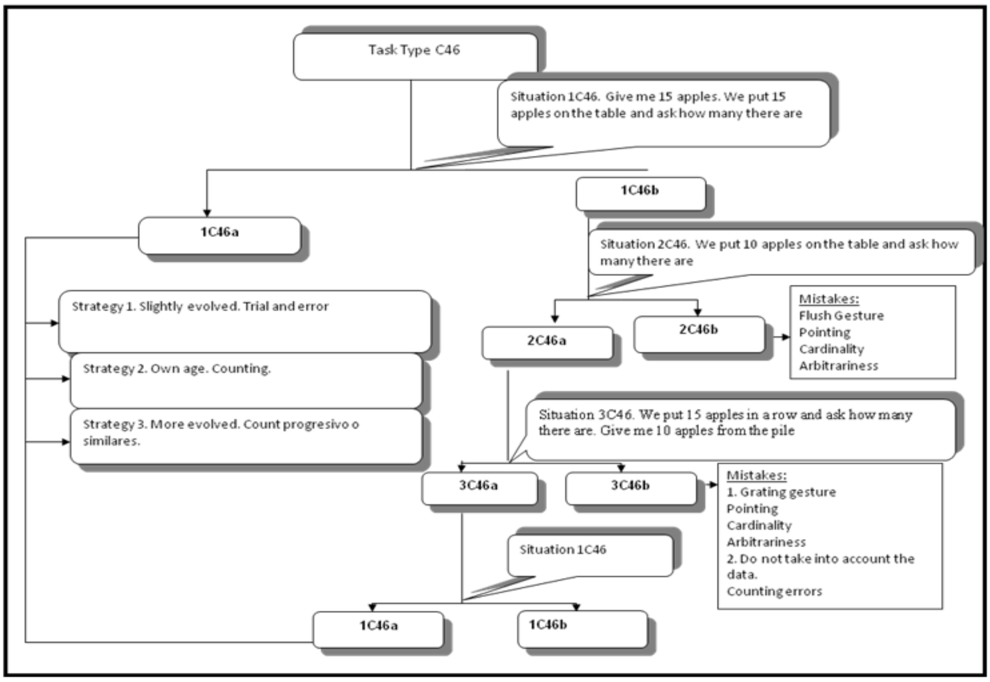

Figure 8. Task to diagnose the cardinal in schoolchildren aged 4 years and a half to 5 years


Figure 9. Do not stop at 15 and keep counting

Ur (4, 6) is a girl who is not in the level corresponding to her age because she does not take into account the data when she is more than 10. So when we say that we of 15 letters begins to count and does not stop in 15 continues counting until the experimenter stops it. However when we are asked to do 9-card counts correctly, therefore this girl would not be in the numberable string level in the sequence 1-15 of the sequence but if she dominates the sequence in this level of countable string in the section 1-10. Then this girl would be in the stage corresponding to 4.0-4.6 (4 years to 4 ½ years).

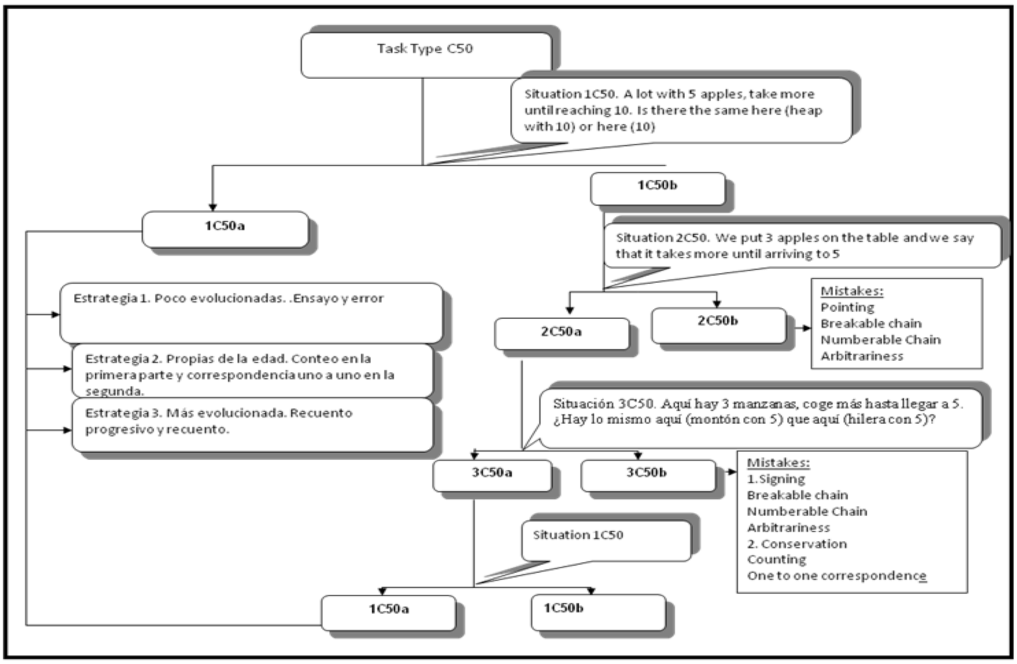

Figure 10. Task to diagnose the cardinal in schoolchildren aged 5 years to 5 ½ years

Figure 11. A heap is pointed out with 5 apples and asked to take more until it reaches 10

Pe (5: 3) is a child who, faced with the question "here are 5 apples, take more until 10" does not take into account that in that pile there are 5 apples and pick up a pile with 10 apples. This child does not use the unbroken string level because it starts counting from 1, in addition this child does not consider that a set can be formed by parts and that the meeting of those parts gives the whole, since the pile of 5 apples already forms part of the new set of 10 apples.

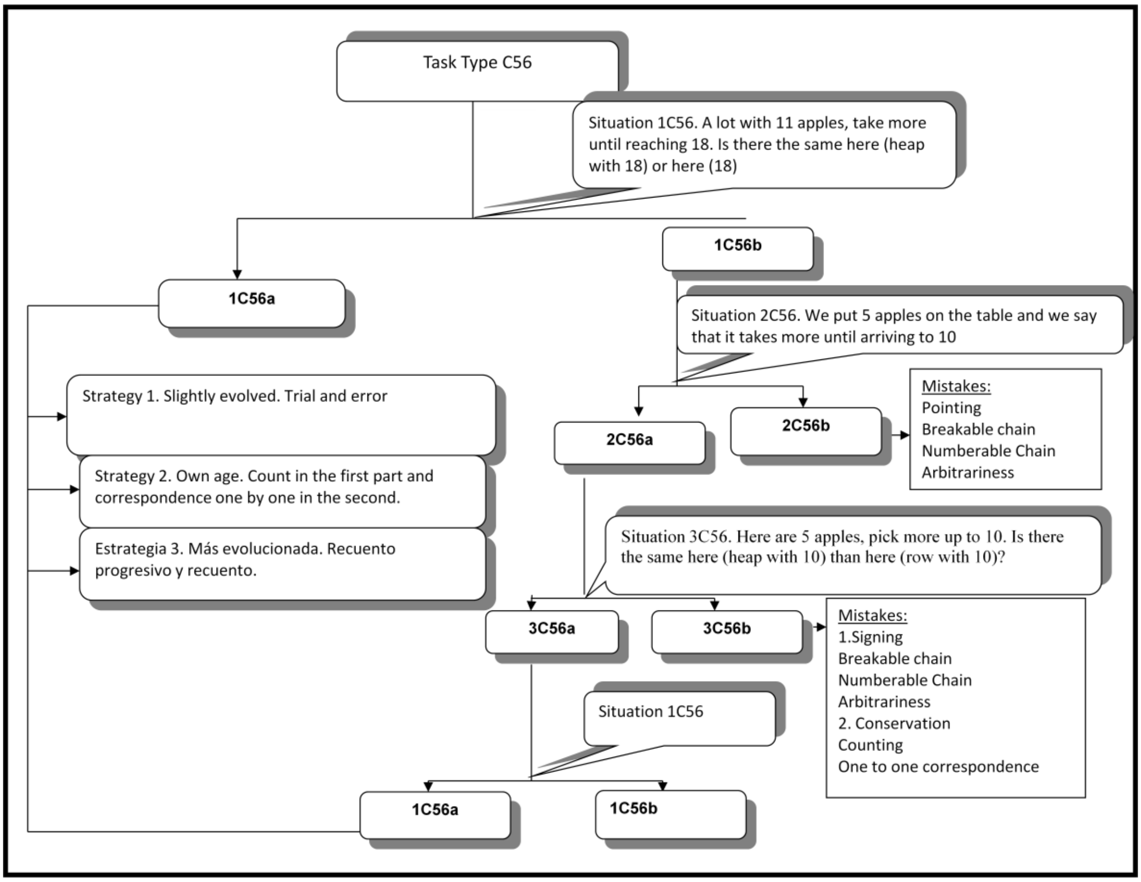

Figure 12. Task to diagnose cardinal in schoolchildren aged 5 years and a half to 6 years

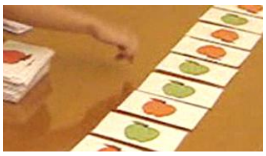

Figure 13. Child counting to find out that there is the same in a pile with 18 cards that 18 cards placed in row

To (5.7). It is a 5 year and 7 month old boy who successfully completes the task corresponding to his age because before the task "here are 11 blocks, take more until reaching 18" count from 11 to get a set with 18 letters, and then to the task "Is there the same here (the experimenter points out the pile of 18 letters) than here (18 letters in a row)," the child counts the letters placed in a row and says that there are 18 and that therefore there are the same cards in the two sets. Use an evolved counting and counting strategy. The cardinal number after counting becomes operative since it is through the number how he comes to the conclusion that the two sets are equal.
This child will pass the tasks corresponding to the period 6.0 and 6.6.

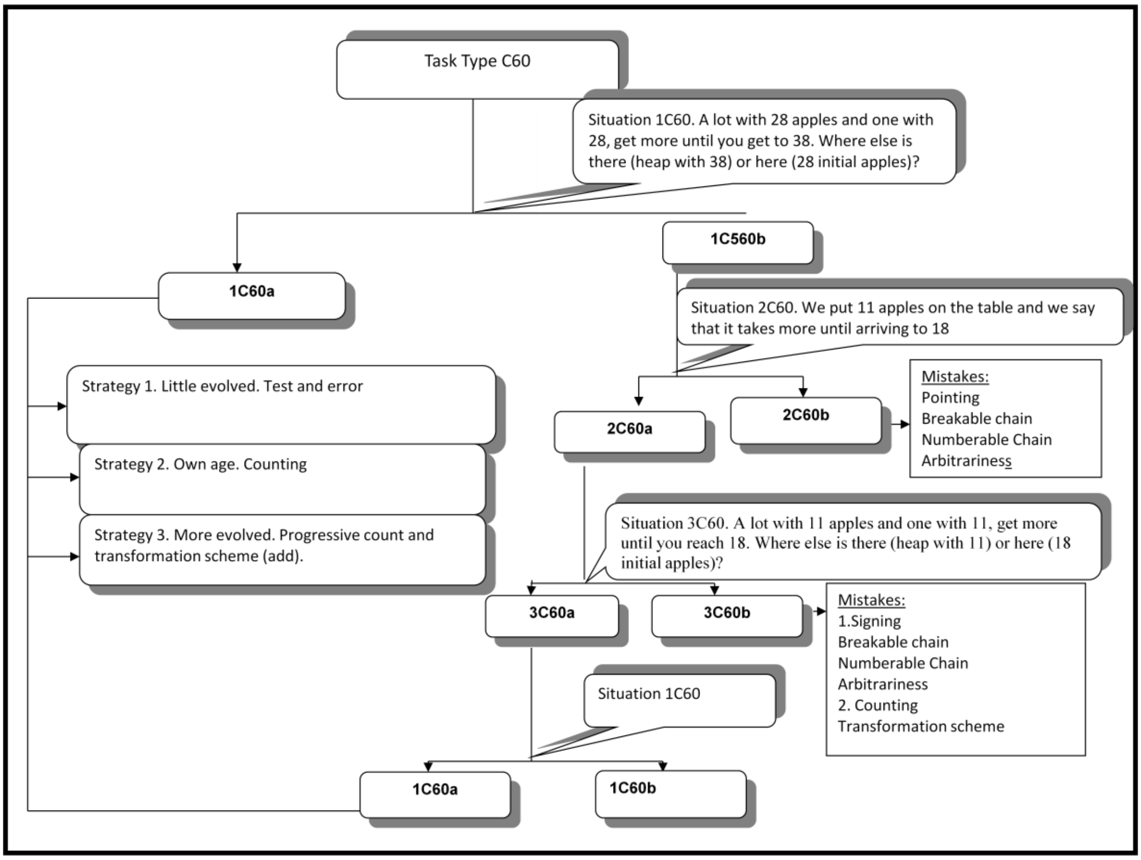

Figure 14. Task to diagnose cardinal in schoolchildren aged 6 years to 6 years and a half

To (5.7). The boy from 28 counts until he reaches 38 cards in the pile, and then comparing that pile with the other where there were 28 cards says there is more in the 38 because there he has added cards. The strategy followed in the first part is progressive counting and in the second uses the transformation scheme is therefore a highly evolved strategy.

II. CONCLUSIONS / SYNTHESIS

There are adequate tasks to diagnose cardinal thinking in schoolchildren aged 3 to 7 years. The design of the tasks is based on the development of the number in the child according to the information processing model in which the counting action is the primary concept that generates the conception of the number in the child.

The tasks have been designed from the conception of cardinal number as the number of elements that has a set.

Taking into account the evolutionary thinking of the child in the act of counting we have been able to analyze both the strategies followed by the students to successfully solve the task corresponding to their age and to analyze the errors that have made the children do not surpass the task corresponding to their age, thus managing to diagnose the cardinal thinking of schoolchildren.

This work is a great help in making a suitable didactic treatment of the cardinal number. Through the tasks of diagnosis we can see the strategies and mistakes of the students and based on that act. For example, for children who make the mistake of not counting the data when we say "that pile give me 9 letters" we can propose activities with correspondence schemes and "as many as" would be: "There we have that group of children (9 boys and girls) and here are a lot of letters. You have to take from that pile the number of letters appropriate so that each child has a letter and not over or missing any. "With this activity the school has to count the number of children (given the whole determine its cardinal) and then use that as a data to take a set of 9 cards. With the slogan of handing out a letter to each boy and girl we get that he does not make the mistake of not taking into account the data.

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