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Infinite Sequences of Primes of Form 4n-1 and 4n+1

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ABSTRACT: *In this work, the author builds a search algorithm for large Primes. It is shown that the number constructed by this algorithm are integers not representable as a sum of two squares. Specified one note of Fermat. Namely, we prove that there are infinitely many numbers of Fermat. It is determined that the first number of Fermat exceeding the number* $2^{2^4}+1$ *satisfies the inequality* $n \geq 17$ *.*
Keywords: *binary mathematical statement, axiom of descent, prime number, number of Fermat.*

I. INTRODUCTION

Leonard Euler attempted to prove one of the most elegant notes Fermat – theorem on Prime numbers [1,73]. All primes except 2 are divided into number, representable in the form $4n+1$, and numbers representable in the form $4n-1$, where n is some integer. Fermat's theorem about Prime numbers claims that Prime numbers of the first group is always representable as a sum of two squares, while primes of the second group never in the form of a sum of two squares is not representable. This property of Prime numbers is formulated elegantly and simply, but all attempts to prove that it has any Prime number $p \neq 2$ encounter considerable difficulties. In 1749, after seven years of work and almost a hundred years after the death of Fermat, Euler managed to prove this theorem on Prime numbers [1,73]. Since this statement is obviously binary [2], we also easily proved using the axiom of descent [3].

II. ALGORITHM OF SEARCH THE LARGE PRIME NUMBERS

In this paper we build an algorithm of search the large Prime numbers and using the axiom of descent prove that the Prime numbers obtained by the algorithm are Prime number of the second group, i.e., not representable as a sum of two squares.

Theorem 1. If n is a Prime number, $n \geq 3$, then 2^n+1 is the product of $3p$, where p is a Prime number of the form $4n-1$, i.e., not representable as a sum of two squares.

Proof. 1. If $n=3$, then $2^n+1=9=3\cdot 3$ and $3=4\cdot 1-1$ or $3=4\cdot n_1-1$, where $n_1=1$. 2. If $n=5$, then $2^n+1=33=3\cdot 11$ and $11=4\cdot 3-1$ or $11=4\cdot n_2-1$, where $n_2=3$. We suppose now $n=p_k$ and $2^{p_k}+1=3\cdot q_k$ and $q_k=4n_k-1$ a Prime number, and for $n=p_{k+1}$ is the place of $2^n+1=3q_{k+1}$ and q_{k+1} is not a number of the form $4n-1$ or is not a Prime number. Then, by the axiom of descent [2] q_k is not a number of the form $4n-1$ or is not a Prime number, which contradict the inductive assumption. The obtained contradiction prove the assertion of the theorem.

The proved theorem to us delivers an algorithm of search of large Prime numbers. If p is a Prime number, $p>3$, then

$$2^p+1=3\cdot q \text{ , where } q>p, q=\frac{2^p+1}{3}=\frac{2^p+1}{2^{\log_2 3}}>2^{p-\log_2 3} .$$

III. PRIME NUMBERS OF FERMAT

In Fermat's notes there is a statement [4,11], that all numbers of a type $2^{2^n}+1$ are Prime but Fermat is the statement has accompanied with a mark that he has no him the satisfactory proof. With some specification of this statement easy with the help of the axiom of descent can be proved that the sequence of integers $2^{2^n}+1, n=1,2,3,4,...$ contains infinitely many Prime numbers. Prime numbers of type $2^{2^n}+1$, usually called [5,35] by numbers Fermat.

Theorem 2. All the numbers in sequence $2^{2^n}+1, n=1,2,3,4,...$ are numbers of the form $4n+1$ and representable as a sum of two squares.

Proof. 1. If $n = 1$, then $2^{2^1} + 1 = 5 = 4 \cdot 1 + 1 = 2^2 + 1^2$. 2. If $n = 2$, then $2^{2^2} + 1 = 17 = 4 \cdot 4 + 1 = 4^2 + 1^2$. 3. If $n = 3$, then $2^{2^3} + 1 = 257 = 4 \cdot 64 + 1 = 16^2 + 1^2$ 4. If $n = 4$, then $2^{2^4} + 1 = 65537 = 4 \cdot 16384 + 1 = 256^2 + 1^2$. If $n \geq 5$, then $2^{2^n} + 1 = 2^2 \cdot 2^{2^n - 2} + 1 = (2 \cdot 2^{\frac{2^{n-2}}{2}})^2 + 1^2$.

Theorem 3. The sequence of integers $2^{2^n} + 1, n1,2,3,4,...$ contains infinitely many Prime numbers.

Proof. From theorem 2 it follows that all Prime in the sequence $2^{2^n} + 1, n = 1,2,3,...$ are Prime numbers representable as a sum of two squares. For $n = 5$, as shown by Euler [4,11], $2^{2^5} + 1$ is a composite number.It can be shown that $2^{2^5} + 1 = 4294967297 = 6700417 \cdot 641$, where 6700417 and 641 are Prime numbers. We will assume that $2^{2^n} + 1$ for $n_1 = 1, n_2 = 2, n_3 = 3, n_4 = 4, n_5$ the numbers $2^{2^{n_i}} + 1, i = \overline{1,5}$, are Prime numbers, and for any $n_6 > n_5$ the number $2^{2^{n_6}} + 1$ is not a Prime number. Then, by the axiom of descent $2^{2^{n_5}} + 1$ is not a Prime number, and this contradicts the inductive assumption. The obtained contradiction proves the assertion of the theorem. We will notice that according to [5,38] $n_5 \geq 17$.

IV. CONCLUSIONS

1. We constructed a search algorithm for large Primes.
2. We have shown that all the numbers constructed by this algorithm are integers not representable as a sum of two squares.
3. We specified Fermat's statement about the numbers of type $2^{2^n} + 1$:

a) we have shown that all numbers of type $2^{2^n} + 1$ are numbers of the form $4n + 1$;
b) we showed that among these numbers there are an infinite number of Prime numbers;

c) we showed that the first Prime number of the type $2^{2^n} + 1$, superior $2^{2^4} + 1$ satisfies the inequality $n \geq 17$.

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