International Journal of Humanities and Social Science Invention
ISSN (Online): 2319 – 7722, ISSN (Print): 2319 – 7714
www.ijhssi.org ||Volume 6 Issue 9||September. 2017 || PP.63-71

Fuzzy Generalized Semi Preregular Closed Sets in Fuzzy Topological Spaces

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Abstract: *In this paper, a new class of sets called fuzzy generalized semi preregular (fgspr) closed sets and open sets in fuzzy topological spaces are introduced and its properties are studied. Further, fgspr-closure, fgspr-interior and fgspr-frontier concepts are also defined and investigated. As an application of this set, we also introduced the notion of fuzzy spaces with new kinds of separation axioms, namely, fuzzy semi preregular $T_{1/2}$ spaces and fuzzy semi preregular $T^*_{1/2}$ spaces and characterize them.*

Keywords: *fuzzy generalized semi preregular closed sets, fuzzy generalized semi preregular open sets, fgspr-closure, fgspr-interior and fgspr-frontier.*

Date of Submission: 11-09-2017 Date of acceptance: 25-09-2017

I. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [19]. Subsequently, several authors have applied various basic concepts from general topology to fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology introduced by Chang [6]. The concept of generalized semi preregular closed sets was introduced and its properties were studied by Govindappa Navalagi et al [8] in 2010.

In this paper fuzzy generalized semi preregular closed set is introduced and its properties are studied. Further, fgspr-closure, fgspr-interior and fgspr-frontier are also defined and investigated. As an application of these fuzzy sets, two new spaces namely fuzzy semi preregular $T_{1/2}$ space and fuzzy semi preregular $T^*_{1/2}$ space are introduced and studied in fuzzy topological spaces. It is observed that every fuzzy semi preregular $T^*_{1/2}$ space is fuzzy semi preregular $T_{1/2}$ space and also fuzzy $T_{1/2}$ space.

II. Preliminaries

Let X, Y and Z be fuzzy sets. Throughout this paper (X,τ), (Y,σ) and (Z,η) (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms are assumed unless explicitly stated. Let us recall the following definitions which we shall require later.

Definition 2.1: A fuzzy set λ in a fuzzy topological space (X,τ) is called

(1) a fuzzy preopen set [4] if $\lambda \leq \mathrm{int}(\mathrm{cl}(\lambda))$ and a fuzzy preclosed set if $\mathrm{cl}(\mathrm{int}(\lambda)) \leq \lambda$.
(2) a fuzzy semi-open set [1] if $\lambda \leq \mathrm{cl}(\mathrm{int}(\lambda))$ and a fuzzy semi-closed set if $\mathrm{int}(\mathrm{cl}(\lambda)) \leq \lambda$.
(3) a fuzzy semi-preopen set[16]if $\lambda \leq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(\lambda)))$ and a fuzzy semi-preclosed set if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(\lambda))) \leq \lambda$.
(4) a fuzzy pre semi-open set [11] if $\lambda \leq \mathrm{int}(\mathrm{cl}(\mathrm{int}(\lambda)))$ and a fuzzy pre semi-closed set if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(\lambda))) \leq \lambda$.
(5) a fuzzy α-open set [4] if $\lambda \leq \mathrm{int}(\mathrm{cl}(\mathrm{int}(\lambda)))$ and a fuzzy α-closed set if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(\lambda))) \leq \lambda$.
(6) a fuzzy regular open set [1] if $\mathrm{int}(\mathrm{cl}(\lambda)) = \lambda$ and a fuzzy regular closed set if $\mathrm{cl}(\mathrm{int}(\lambda)) = \lambda$.

Definition 2.2: A fuzzy set λ in a fuzzy topological space (X,τ) is called

(1) a fuzzy generalized closed set (briefly, fg-closed) [2] if $\mathrm{cl}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy open set in X.
(2) a generalized fuzzy semi-closed set (briefly, gfs-closed) [3] if $\mathrm{scl}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy semi-open set in X.
(3) a fuzzy regular generalized closed set (briefly, frg-closed) [12] if $\mathrm{cl}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy regular open set in X.
(4) a fuzzy semi generalized closed set (briefly, fsg-closed) [10] if $\mathrm{scl}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy semi-open set in X.
(5) a fuzzy generalized semi-closed set (briefly, fgs-closed) [14] if $\mathrm{scl}(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy open set in X.

(6) a fuzzy generalized pre closed set (briefly, fgp-closed) [7] if $pcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy open set in X.
(7) a fuzzy generalized semi-pre closed set (briefly, fgsp-closed) [13] if $spcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy open set in X.
(8) a fuzzy semi-pre generalized closed set (briefly, fspg-closed) [15] if $spcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy semi-pre open set in X.
(9) a fuzzy generalized preregular closed set (briefly, fgpr-closed) [18] if $pcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy regular open set in X.

Definition 2.3: A fuzzy topological space $(X, \tau)$is said to be
(1) a fuzzy $T_{1/2}$ -space [2] if every fg-closed is fuzzy closed.
(2) a fuzzy semi $T_{1/2}$ -space [10] if every fsg-closed is fuzzy semi closed.
(3) a fuzzy pre $T_{1/2}$ -space [5] if every fpg-closed is fuzzy pre closed.
(4) a fuzzy semi pre $T_{1/2}$-space [13] if every fgsp-closed is fuzzy semi-preclosed.
(5) a fuzzy semi pre $T_{1/3}$-space [13] if every fgsp-closed is fuzzy pre semi closed.
(6) a fuzzy pre semi $T_{1/2}$-space [13] if every fps-closed is fuzzy semi-preclosed.

III. Fuzzy Generalized Semi Preregular Closed Sets

In this section, a new class of fuzzy generalized closed sets called a fuzzy generalized semi preregular closed set is defined and study its properties.

Definition 3.1: A fuzzy set λ of a fuzzy topological space (X, τ) is called a fuzzy generalized semi preregular closed set (briefly, fgspr-closed) if $spcl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$ and μ is a fuzzy regular open set in (X, τ).

By FGSPRC we mean the family of all fgspr-closed subsets of the space (X, τ).

Theorem 3.2: Every fuzzy closed set is a fgspr-closed set.

Proof: Let λ be a fuzzy closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy closed set is a fuzzy semi-pre closed set, $spcl(\lambda) \leq cl(\lambda) = \lambda \leq \mu$. (i.e) $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.3: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.2)} and λ_2 = {(a, 0.2), (b, 0.4), (c, 0.1)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, 1\}$, then λ_2 is a fgspr-closed set but not a fuzzy closed set in X.

Theorem 3.4: Every fuzzy preclosed set is a fgspr-closed set in X.

Proof: Let λ be a fuzzy preclosed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy preclosed is a fuzzy semi preclosed, $spcl(\lambda) \leq pcl(\lambda) = \lambda \leq \mu$. (i.e) $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.5: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)}, λ_2 = {(a, 0.1), (b, 0.3), (c, 0.2)} and λ_3 = {(a, 0.2), (b, 0.5), (c, 0.3)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$, then λ_3 is a fgspr- closed set but not a fuzzy preclosed set in X.

Theorem 3.6: Every fuzzy α-closed set is a fgspr-closed set in X.

Proof: Let λ be a fuzzy α-closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy α-closed set is a fuzzy pre closed set, $spcl(\lambda) \leq pcl(\lambda) \leq \alpha cl(\lambda) = \lambda \leq \mu$. (i.e) $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.7: In example 3.5, the fuzzy set λ_3 is a fgspr-closed set but not a fuzzy α-closed set in X.

Theorem 3.8: Every fuzzy regular generalized closed set is a fgspr-closed set in X.

Proof: Let λ be a fuzzy regular generalized closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy closed set is fuzzy semi pre closed set, $spcl(\lambda) \leq cl(\lambda) \leq \mu$. (i.e) $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.9: In example 3.5, the fuzzy set λ_3 is a fgspr-closed set but not a fuzzy regular generalized closed set in X.

Theorem 3.10: Every fuzzy generalized semi-pre closed set is a fgspr-closed set in X.

Proof: Let λ be a fuzzy generalized semi-pre closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy regular open set is fuzzy open set, $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.11: Let X = {a, b, c}, α ={(a, 1), (b, 0), (c, 0)}, β = {(a, 0), (b, 1), (c, 0)}, γ ={(a, 1), (b, 1), (c, 0)} and η = {(a, 1), (b, 0), (c, 1)} be fuzzy sets of X. Let $\tau = \{0, \alpha, \beta, \gamma, \eta, 1\}$, then α is a fgspr-closed set but not a fgsp-closed set in X.

Theorem 3.12: Every fgpr-closed set is a fgspr-closed set in X.
Proof: Let λ be a fgpr-closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since λ is fgpr-closed set, $pcl(\lambda) \leq \mu$. Since $spcl(\lambda) \leq pcl(\lambda)$, for any fuzzy set λ, $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.13: In example 3.5, the fuzzy set λ_3 is a fgspr-closed set but not a fgpr-closed set in X.

Theorem 3.14: Every fuzzy generalized preclosed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy generalized preclosed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy generalized preclosed set is fgpr-closed [12], by Theorem 3.12 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.15: In example 3.5, the fuzzy set λ_3 is a fgspr-closed set but not a fuzzy generalized preclosed set in X.

Theorem 3.16: Every fuzzy generalized closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy generalized closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy generalized closed set is fgp-closed [6], by Theorem 3.14 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.17: In example 3.5, the fuzzy set λ_3 is a fgspr-closed set but not a fuzzy generalized closed set in X.

Theorem 3.18: Every fuzzy semi-pre closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy semi-pre closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy semi-pre closed set is fgsp-closed set, by Theorem 3.10 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.19: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)}, λ_2 = {(a, 0.1), (b, 0.3), (c, 0.2)} and λ_3 = {(a, 0.2), (b, 0.5), (c, 0.4)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$, then λ_3 is a fgspr-closed set but not a fuzzy semi-pre closed set in X

Theorem 3.20: Every fuzzy pre semi closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy pre semi closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy pre semi closed set is fgsp-closed set, by Theorem 3.10 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.21: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)}, λ_2 = {(a, 0.1), (b, 0.5), (c, 0.4)} and λ_3 = {(a, 0.1), (b, 0.3), (c, 0.2)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, \lambda_3, 1\}$, then λ_2 is a fgspr-closed set but not a fuzzy pre semi closed set in X.

Theorem 3.22: Every fuzzy semi-closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy semi-closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy semi-closed set is fuzzy semi-preclosed set, by Theorem 3.18 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.23: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)} and λ_2 = {(a, 0.2), (b, 0.5), (c, 0.4)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, 1\}$, then λ_2 is a fgspr-closed set but not a fuzzy semi-closed set in X.

Theorem 3.24: Every fuzzy semi-pre-generalized closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy semi-pre-generalized closed set in a fuzzy topological space (X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy semi-preclosed set is a fuzzy semi-pre generalized closed set [15] and every fuzzy semi-pre-generalized closed set is a fgsp-closed set, by Theorem 3.10 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.25: In example 3.11, the fuzzy set α is a fgspr-closed set but not a fuzzy semi-pre-generalized closed set in X.

Theorem 3.26: Every fuzzy regular closed set is a fgspr-closed set in X.
Proof: Let λ be a fuzzy regular closed set in a fuzzy topological space(X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy regular closed set is a fuzzy closed set, by Theorem 3.2 $spcl(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.27: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)}, λ_2 = {(a, 0.1), (b, 0.3), (c, 0.2)} and λ_3 = {(a, 0.1), (b, 0.5), (c, 0.4)} be fuzzy sets of X. Let $\tau = \{0, \lambda_1, \lambda_2, 1\}$, then λ_3 is a fgspr-closed set but not a fuzzy regular closed set in X.

Theorem 3.28: Every fuzzy generalized semi-closed set is a fgspr-closed set in X.

Proof: Let λ be a fuzzy generalized semi-closed set in a fuzzy topological space(X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy semi-closed set is a fuzzy generalized semi-closed set [6] and every fuzzy generalized semi-closed set is a fgsp-closed set, by Theorem 3.10 $\mathrm{spcl}(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.29: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.4)}, λ_2 = {(a, 0.1), (b, 0.3), (c, 0.2)} and λ_3 = {(a, 0), (b, 0.2), (c, 0.1)} be fuzzy sets of X. Let τ = {0, λ_1, λ_2, 1}, then λ_3 is a fgspr-closed set but not a fuzzy generalized semi-closed set in X.

Theorem 3.30: Every generalized fuzzy semi-closed set is a fgspr-closed set in X.

Proof: Let λ be a generalized fuzzy semi-closed set in a fuzzy topological space(X, τ). Suppose that $\lambda \leq \mu$, where μ is fuzzy regular open in X. Since every fuzzy semi-closed set is a generalized fuzzy semi-closed set [15] and every generalized fuzzy semi-closed set is a fgsp-closed set, by Theorem 3.10 $\mathrm{spcl}(\lambda) \leq \mu$. Hence λ is a fgspr-closed set in X.

The following example shows that the converse of the above theorem is not true.

Example 3.31: In example 3.27, the fuzzy set λ_3 is a fgspr-closed set but not a generalized fuzzy semi-closed set in X.

Theorem 3.32: In a fuzzy topological space(X, τ), if a fuzzy set λ is both fuzzy regular open and fgspr-closed, then λ is fuzzy semi-pre closed.

Proof: Suppose a fuzzy set λ of a fuzzy topological space (X, τ) is both regular open and fgspr-closed. Let $\lambda \leq \lambda$, where λ is fuzzy regular open in X. This implies that $\mathrm{spcl}(\lambda) \leq \lambda$, since λ is a fgspr-closed set. Also we have, $\lambda \leq \mathrm{spcl}(\lambda)$, which implies that $\mathrm{spcl}(\lambda) = \lambda$. Hence λ is a fuzzy semi-pre closed set in X.

Theorem 3.33: If a fuzzy set λ is fgspr-closed in X and $\mathrm{spcl}(\lambda) \wedge \big(1 - \mathrm{spcl}(\lambda)\big) = 0$ then there is no non zero fuzzy regular closed set μ such that $\mu \leq \mathrm{spcl}(\lambda) \wedge (1 - \lambda)$.

Proof: Suppose μ is any fuzzy regular closed set in X such that $\mu \leq \mathrm{spcl}(\lambda) \wedge (1 - \lambda)$. Now $\mu \leq 1 - \lambda$, this implies that $\lambda \leq 1 - \mu$, where $1 - \mu$ is fuzzy regular open in X. Then $\mathrm{spcl}(\lambda) \leq 1 - \mu$, as λ is fgspr-closed set. This implies that $\mu \leq 1 - \mathrm{spcl}(\lambda)$. Thus $\mu \leq \mathrm{spcl}(\lambda)$ and $\mu \leq 1 - \mathrm{spcl}(\lambda)$. Therefore $\mu \leq \mathrm{spcl}(\lambda) \wedge (1 - \mathrm{spcl}(\lambda)) = 0$. Thus $\mu = 0$. Hence the result is proved.

Theorem 3.34: If a fuzzy set λ is fgspr-closed in X such that $\lambda \leq \mu \leq \mathrm{spcl}(\lambda)$, then μ is also a fgspr-closed set in X.

Proof: Let γ be a fuzzy regular open set in X such that $\mu \leq \gamma$. Then $\lambda \leq \gamma$. Since λ is fgspr-closed set, then by definition, $\mathrm{spcl}(\lambda) \leq \mu$. By hypothesis, $\mu \leq \mathrm{spcl}(\lambda)$, so $\mathrm{spcl}(\mu) \leq \mathrm{spcl}\big(\mathrm{spcl}(\lambda)\big) = \mathrm{spcl}(\lambda) \leq \mu \leq \gamma$. (i.e) $\mathrm{spcl}(\mu) \leq \gamma$. Hence μ is a fgspr-closed set in X.

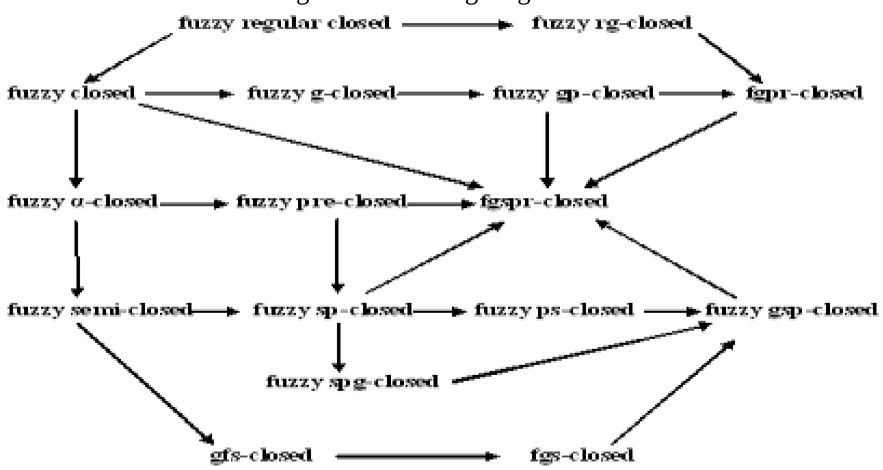
Theorem 3.35: The union of any two fgspr-closed sets is a fgspr-closed set.

Proof: Let λ and μ be fgspr-closed sets in a fuzzy topological space X. To prove that $\lambda \vee \mu$ is a fgspr-closed set. Let $\lambda \vee \mu \leq \gamma$, where γ be fuzzy regular open in X. Then $\lambda \leq \gamma$, $\mu \leq \gamma$ and so $\mathrm{spcl}(\lambda) \leq \gamma$, $\mathrm{spcl}(\mu) \leq \gamma$ as λ and μ are fgspr-closed sets. This implies that $\mathrm{spcl}(\lambda) \vee \mathrm{spcl}(\mu) \leq \gamma$. (i.e) $\mathrm{spcl}(\lambda \vee \mu) \leq \gamma$. Hence $\lambda \vee \mu$ is a fgspr-closed set in X.

Remark 3.36: The intersection of any two fgspr-closed sets in a fuzzy topological space X is not necessarily fgspr-closed as seen from the following example.

Example 3.37: Let X = {a, b, c}, λ_1 = {(a, 0.4), (b, 0.3), (c, 0.5)}, λ_2 = {(a, 0.3), (b, 0.9), (c, 0.5)} and λ_3 = {(a, 0.7), (b, 0.4), (c, 0.8)} be fuzzy sets of X. Let τ = {0, λ_1, 1}, then λ_2 and λ_3 are fgspr-closed set but $\lambda_2 \wedge \lambda_3$ is not a fgspr-closed set in X.

Remark 3.38: From the above results we get the following diagram.



IV. Fuzzy Generalized Semi Preregular Open Sets

Definition 4.1: A fuzzy set λ of a fuzzy topological space (X, τ) is called fuzzy generalized semi preregular open (briefly fgspr-open) set if its complement 1 – λ is fgspr-closed set.

Theorem 4.2: A fuzzy set λ of a fuzzy topological space X is fgspr-open iff $\mu \leq \text{spint}(\lambda)$, whenever μ is fuzzy regular closed set and $\mu \leq \lambda$.

Proof: Suppose λ is fgspr-open set in a fuzzy topological space X. Then 1 – λ is fgspr-closed set in X. Let μ be fuzzy regular closed in X and $\mu \leq \lambda$. Then $1-\lambda \leq 1-\mu$, where $1-\mu$ is fuzzy regular open. Since 1 – λ is a fgspr-closed set, we have $\text{spcl}(1-\lambda) \leq 1-\mu$. This implies that $1-(1-\mu) \leq 1-\text{spcl}(1-\lambda)$. (i.e) $\mu \leq \text{spint}(\lambda)$ as $\text{spcl}(1-\lambda) = 1-\text{spint}(\lambda)$ (by Lemma 1.5 [11]).

Conversely, assume that $\mu \leq \text{spint}(\lambda)$, whenever $\mu \leq \lambda$ and μ is fuzzy regular closed set in a fuzzy topological space X. Let $1-\lambda \leq \gamma$, where γ is fuzzy regular open set in X. Then $1-\gamma \leq \lambda$, where $1-\gamma$ is fuzzy regular closed set in X. This implies that, $1-\gamma \leq \text{spint}(\lambda)$. (i.e) $1-\text{spint}(\lambda) \leq 1-(1-\gamma)$. (i.e) $\text{spint}(1-\lambda) \leq \gamma$. Hence 1 – λ is fgspr-closed set and so λ is fgspr-open set in X.

Theorem 4.3: Every fuzzy open(fp-open, fα-open, frg-open, fgsp-open, fgpr-open, fgp-open, fg-open, fsp-open, fps-open, fs-open, fspg-open, fr-open, fgs-open, gfs-open) set is a fgspr-open set.

Proof: Let λ be fuzzy open(fp-open, fα-open, frg-open, fgsp-open, fgpr-open, fgp-open, fg-open, fsp-open, fps-open, fs-open, fspg-open, fr-open, fgs-open, gfs-open) set in a fuzzy topological space(X, τ). Then 1 – λ is fuzzy closed set in X. And so 1 – λ is fgspr-closed set in X by Theorem 3.2(3.4, 3.6, 3.8, 3.10, 3.12, 3.14, 3.16, 3.18, 3.20, 3.22, 3.24, 3.26, 3.28, 3.30). Hence λ is a fgspr-open set in X.

The following examples show that the converse of the above theorem is not true.

Example 4.4: Let X = {a, b, c}, λ_1 = {(a, 0.8), (b, 0.5), (c, 0.3)} and λ_2 = {(a, 0.7), (b, 0.5), (c, 0.2)} be fuzzy sets of X. Let τ = {0, λ_2, 1}, then λ_1 is a fgspr-open set but not a fuzzy open set in X.

Example 4.5: Let X = {a, b, c}, λ_1 = {(a, 0), (b, 0.4), (c, 0.2)} and λ_2 = {(a, 0.7), (b, 0.5), (c, 0.2)} be fuzzy sets of X. Let τ = {0, λ_2, 1}, then λ_1 is a fgspr-open set but not a fp-open set and a fα-open set in X.

Example 4.6: Let X = {a, b, c}, λ_1 = {(a, 0.5), (b, 0.4), (c, 1)} and λ_2 = {(a, 0.7), (b, 0.5), (c, 0.2)} and λ_3 = {(a, 0.6), (b, 1), (c, 0.4)} be fuzzy sets of X. Let τ = {0, λ_1, λ_2, 1}, then λ_3 is a fgspr-open set but not a frg-open set in X.

Example 4.7: Let X = {a, b, c}, α = {(a, 1), (b, 0), (c, 0)}, β = {(a, 0), (b, 1), (c, 0)}, γ = {(a, 1), (b, 1), (c, 0)}, η = {(a, 1), (b, 0), (c, 1)} and μ = {(a, 0), (b, 1), (c, 1)} be fuzzy sets of X. Let τ = {0, α, β, γ, η, 1}, then μ is a fgspr-open set but not a fgsp-open set, a fgpr-open set, a fgp-open set, fg-open set, a fspg-open set, a fgs-open set and a gfs-open set in X.

Example 4.8: Let X = {a, b, c}, λ_1 = {(a, 0.3), (b, 0.5), (c, 0.8)} and λ_2 = {(a, 0), (b, 0.4), (c, 0.2)} be fuzzy sets of X. Let τ = {0, λ_1, 1}, then λ_2 is a fgspr-open set but not a fsp-open set, a fps-open, a fs-open set and a fuzzy regular open set in X.

Theorem 4.9: If $\text{spint}(\lambda) \leq \mu \leq \lambda$ and if λ is fgspr-open, then μ is a fgspr-open set in a fuzzy topological space X.

Proof: We have $\text{spint}(\lambda) \leq \mu \leq \lambda$, then $1-\lambda \leq 1-\mu \leq 1-\text{spint}(\lambda)$. (i.e) $1-\lambda \leq 1-\mu \leq \text{spcl}(1-\lambda)$ and since 1 – λ is a fgspr-closed set and by Theorem 3.30, we have $1-\mu$ is a fgspr-closed set in X. Hence μ is a fgspr-open set in X.

Theorem 4.10: The intersection of any two fgspr-open sets is a fgspr-open set.

Proof: Let λ and μ be fgspr-open set in a fuzzy topological space X. To prove that $\lambda \wedge \mu$ is fgspr-open set. Let $\gamma \leq \lambda \wedge \mu$, where γ is fuzzy regular closed in X. Then $\gamma \leq \lambda$, $\gamma \leq \mu$ and so $\gamma \leq \text{spint}(\lambda)$, $\gamma \leq \text{spint}(\mu)$ as λ and μ are fgspr-open sets. This implies that $\gamma \leq \text{spint}(\lambda) \wedge \text{spint}(\mu) = \text{spint}(\lambda \wedge \mu)$. (i.e) $\gamma \leq \text{spint}(\lambda \wedge \mu)$. Hence $\lambda \wedge \mu$ is a fgspr-open set in X.

Remark 4.11: The union of any two fgspr-open sets in a fuzzy topological space X is not necessarily fgspr-open as seen from the following example.

Example 4.12: Let X = {a, b, c}, λ_1 = {(a, 0.7), (b, 0.5), (c, 0.2)}, λ_2 = {(a, 0), (b, 0.4), (c, 0.5)} and λ_3 = {(a, 0.6), (b, 1), (c, 0.4)} be fuzzy sets of X. Let τ = {0, λ_1, 1}, then λ_2 and λ_3 are fgspr-open sets but $\lambda_2 \vee \lambda_3$ is not a fgspr-open set in X.

Theorem 4.13: If a fuzzy set λ is fgspr-closed set and $\text{spcl}(\lambda) \wedge [1-\text{spcl}(\lambda)] = 0$ then $\text{spcl}(\lambda) \wedge (1-\lambda)$ is a fgspr-open set in X.

Proof: Let λ be fgspr-closed set in a fuzzy topological space X. Let $\text{spcl}(\lambda) \wedge (1-\lambda)$ and μ is fuzzy regular closed set in X. By Theorem 3.31, μ is zero and so $\mu \leq \text{spint}(\text{spcl}(\lambda) \wedge (1-\lambda))$. By Theorem 4.2, $\text{spcl}(\lambda) \wedge (1-\lambda)$ is a fgspr-open set in X.

V. Fgspr-closure and Fgspr-interior

In this section, fgspr-closure (fgspr-cl) and fgspr-interior (fgspr-int) of a fuzzy set is defined as follows.

Definition 5.1: For any fuzzy set λ in any fuzzy topological space,

fgspr-cl(λ) = $\wedge\{\mu : \mu$ is a fgspr-closed set and $\lambda \leq \mu\}$

fgspr-int(λ) = $\vee${μ: μ is a fgspr-open set and $\lambda \geq \mu$}

Theorem 5.2: Let λ be any fuzzy set in a fuzzy topological space(X, τ). Then fgspr-cl($1 - \lambda$) = $1 -$ fgspr-int(λ) and fgspr-int($1 - \lambda$) = $1 -$ fgspr-cl(λ)

Proof: We know that a fgspr-open set $\mu \leq \lambda$ is precisely the complement of a fgspr-closed set $1 - \mu \geq 1 - \lambda$ and thus fgspr-int(λ) = $\vee${μ: μ is a fgspr-open set and $\lambda \geq \mu$} = fgspr-int(λ) = $\vee${$1 - \mu$:$1 - \mu$ is a fgspr-closed set and $1 - \mu \geq 1 - \lambda$} = $1 - \wedge${$1 - \mu$: $1 - \mu$ is a fgspr-closed set and $1 - \mu \geq 1 - \lambda$ } = $1 -$ fgspr-cl($1 - \lambda$). Thus fgspr-cl($1 - \lambda$) = $1 -$ fgspr-int(λ).

Replacing λ by $1 - \lambda$ in the previous result we get, fgspr-cl(λ) = $1 -$ fgspr-int($1 - \lambda$). Thus fgspr-int($1 - \lambda$) = $1 -$ fgspr-cl(λ).

Theorem 5.3: In a fuzzy topological space(X, τ), a fuzzy set λ is fgspr-closed set then λ = fgspr-cl(λ)

Proof: Let λ be a fgspr-closed set in a fuzzy topological space(X, τ). Then we have fgspr-cl(λ) $\leq \lambda$. But $\lambda \leq$ fgspr-cl(λ) always. Therefore λ = fgspr-cl(λ).

Theorem 5.4: In a fuzzy topological space X the following results hold for fgspr-closure.

(i) fgspr-cl(0) = 0, fgspr-cl(1) = 1

(ii) $\lambda \leq$ fgspr-cl(λ) $\leq$ f-cl(λ)

(iii) fgspr-cl(fgspr-cl(λ)) = fgspr-cl(λ)

Proof: Let λ be a fuzzy set in a fuzzy topological space(X, τ).

(i) Obvious.

(ii) Every fuzzy closed set is a fgspr-closed set. By Theorem 3.2, fgspr-cl(λ) $\leq$ f-cl(λ) and by definition 5.1, $\lambda \leq$ fgspr-cl(λ). Hence $\lambda \leq$ fgspr-cl(λ) $\leq$ f-cl(λ).

(iii) By Theorem 5.3, fgspr-cl(fgspr-cl(λ)) = fgspr-cl(λ), since fgspr-cl(λ) is fgspr closed.

Theorem 5.5: In a fuzzy topological space X the following results hold for fgspr-closure.

(i) fgspr-cl(λ) $\leq$ fgspr-cl(μ) if $\lambda \leq \mu$

(ii) fgspr-cl(λ) $\vee$ fgspr-cl(μ) $\leq$ fgspr-cl($\lambda \vee \mu$)

(iii) fgspr-cl($\lambda \wedge \mu$) $\leq$ fgspr-cl(λ) $\wedge$ fgspr-cl(μ)

Proof: Let λ and μ be fuzzy sets in a fuzzy topological space(X, τ).

(i) Since $\lambda \leq \mu$, a fgspr-closed set containing μ, contains λ also. Therefore fgspr-cl(λ) $\leq$ fgspr-cl(μ).

(ii) Let $\lambda \leq \lambda \vee \mu$ and $\mu \leq \lambda \vee \mu$. This implies that fgspr-cl(λ) $\leq$ fgspr-cl($\lambda \vee \mu$) and fgspr-cl(μ) $\leq$ fgspr-cl($\lambda \vee \mu$) by (i). Hence fgspr-cl(λ) $\vee$ fgspr-cl(μ) $\leq$ fgspr-cl($\lambda \vee \mu$).

(iii) Let $\lambda \wedge \mu \leq \lambda$ and $\lambda \wedge \mu \leq \mu$. This implies that fgspr-cl($\lambda \wedge \mu$) $\leq$ fgspr-cl(λ) and fgspr-cl($\lambda \wedge \mu$) $\leq$ fgspr-cl(μ) by (i). Hence fgspr-cl($\lambda \wedge \mu$) $\leq$ fgspr-cl(λ) $\wedge$ fgspr-cl(μ).

Remark 5.6: For any two fuzzy sets λ and μ, fgspr-cl(λ) = fgspr-cl(μ) does not imply that $\lambda = \mu$. This is shown by the following example.

Example 5.7: Let X = {a, b, c}, α = {(a, 0), (b, 0), (c, 1)}, β = {(a, 1), (b, 0), (c, 1)}, λ = {(a, 0), (b, 0), (c, 1)} and μ = {(a, 0), (b, 1), (c, 1)} be fuzzy sets of X. Let τ = {0, α, β, 1}, then fgspr-cl(λ) = fgspr-cl(μ) = 1. It follows that, fgspr-cl(λ) = fgspr-cl(μ) but $\lambda \neq \mu$.

Theorem 5.8: In a fuzzy topological space(X, τ), a fuzzy set λ is fgspr-open iff λ = fgspr-int(λ).

Proof: Let λ be a fgspr-open set in a fuzzy topological space(X, τ). Then we have $\lambda \leq$ fgspr-int(λ). But fgspr-int(λ) $\leq \lambda$ always. Therefore λ = fgspr-int(λ).

Suppose that λ = fgspr-int(λ) and by definition 5.1, fgspr-int(λ) is a fgspr-open set. Then λ is a fgspr-open set in X.

Theorem 5.9: In a fuzzy topological space X the following results hold for fgspr-interior.

(i) fgspr-int(0) = 0, fgspr-int(1) = 1

(ii) f-int(λ) $\leq$ fgspr-int(λ) $\leq \lambda$

(iii) fgspr-int(fgspr-int(λ)) = fgspr-int(λ)

Proof: Let λ be a fuzzy set in a fuzzy topological space(X, τ).

(i) Obvious.

(ii) Every fuzzy open set is a fgspr-open set. By Theorem 4.3, f-int(λ) $\leq$ fgspr-int(λ) and by definition 5.1, fgspr-int(λ) $\leq \lambda$. Hence f-int(λ) $\leq$ fgspr-int(λ) $\leq \lambda$.

(iii) By Theorem 5.8, fgspr-int(fgspr-int(λ)) = fgspr-int(λ), since fgspr-int(λ) is fgspr open.

Theorem 5.10: In a fuzzy topological space X the following results hold for fgspr-interior.

(i) fgspr-int(λ) $\leq$ fgspr-int(μ) if $\lambda \leq \mu$

(ii) fgspr-int(λ) $\vee$ fgspr-int(μ) $\leq$ fgspr-int($\lambda \vee \mu$)

(iii) fgspr-int($\lambda \wedge \mu$) $\leq$ fgspr-int(λ) $\wedge$ fgspr-int(μ)

Proof: Let λ and μ be fuzzy sets in a fuzzy topological space(X, τ).

(i) Since $\lambda \leq \mu$, a fgspr-open set contained in λ is also contained in μ. Therefore fgspr-int(λ) $\leq$ fgspr-int(μ).

(ii) Let $\lambda \leq \lambda \vee \mu$ and $\mu \leq \lambda \vee \mu$. This implies that fgspr-int(λ) $\leq$ fgspr-int($\lambda \vee \mu$) and fgspr-int(μ) $\leq$ fgspr-intl($\lambda \vee \mu$) by (i). Hence fgspr-int(λ) $\vee$ fgspr-int(μ) $\leq$ fgspr-int($\lambda \vee \mu$)

(iii) Let λ ∧ μ ≤ λ and λ ∧ μ ≤ μ. This implies that fgspr-int(λ ∧ μ) ≤ fgspr-int(λ) and fgspr-int(λ ∧ μ) ≤ fgspr-int(μ) by (i). Hence fgspr-int(λ ∧ μ) ≤ fgspr-int(λ) ∧ fgspr-int(μ)

Remark 5.11: For any two fuzzy sets λ and μ, fgspr-int(λ) = fgspr-int(μ) does not imply that λ = μ. This is shown by the following example.

Example 5.12: Let X = {a, b, c}, α = {(a, 0), (b, 0), (c, 1)}, β = {(a, 1), (b, 0), (c, 1)}, λ = {(a, 1), (b, 0), (c, 0)} and μ = {(a, 0), (b, 1), (c, 0)} be fuzzy sets of X. Let τ = {0, α, β, 1}, then fgspr-int(λ) = fgspr-int(μ) = 0. It follows that, fgspr-int(λ) = fgspr-int(μ) but λ ≠ μ.

VI. Fgspr – frontier of a set

Definition 6.1: fgspr-cl(λ) – fgspr-int(λ) is defined to be the fgspr-frontier of λ in a fuzzy topological space (X, τ) and is denoted by fgspr-fr(λ).

Some basic properties of fgspr-fr(λ) are proved in the following:

Theorem 6.2: In a fuzzy topological space X the following results hold

(i) fgspr-cl(λ) = fgspr-int(λ) ∨ fgspr-fr(λ)

(ii) fgspr-cl(λ) ∧ fgspr-fr(λ) = fgspr-fr(λ)

(iii) fgspr-int(λ) ∧ fgspr-fr(λ) = 0

(iv) fgspr-fr(λ) = fgspr-cl(λ) ∧ fgspr-cl(1 – λ)

Proof: By definition of fgspr-fr(λ), we have

(i) fgspr-int(λ) ∨ fgspr-fr(λ) = fgspr-int(λ) ∨ [fgspr-cl(λ) – fgspr-int(λ)] = fgspr-cl(λ).

(ii) fgspr-cl(λ) ∧ fgspr-fr(λ) = fgspr-cl(λ) ∧ [fgspr-cl(λ) – fgspr-int(λ)] = fgspr-cl(λ) – fgspr-int(λ) = fgspr-fr(λ).

(iii) fgspr-int(λ) ∧ fgspr-fr(λ) = fgspr-int(λ) ∧ [fgspr-cl(λ) – fgspr-int(λ)] = 0.

(iv) fgspr-fr(λ) = fgspr-cl(λ) – fgspr-int(λ) = fgspr-cl(λ) ∧ [1 – fgspr-int(λ)] = fgspr-cl(λ) ∧ fgspr-cl(1 – λ) by theorem 5.2.

Theorem 6.3: In a fuzzy topological space X the following results hold

(i) fgspr-fr(λ) = fgspr-fr(1 – λ)

(ii) If λ is fgspr open then λ ∧ fgspr-fr(λ) = 0

(iii) fgspr-fr(λ) = 0 if λ is fgspr-open as well as fgspr-closed.

Proof: By definition of fgspr-fr(λ), we have

(i) fgspr-fr(λ) = fgspr-cl(λ) – fgspr-int(λ) = fgspr-cl(λ) ∧ [1 – fgspr-int(λ)] = fgspr-cl(λ) ∧ fgspr-cl(1 – λ) = fgspr-cl(1 – λ) ∧ [1 – fgspr-int(1 – λ) = fgspr-cl(1 – λ) – fgspr-int(1 – λ) = fgspr-fr(1 – λ) as by theorem 5.2.

(ii) Let λ be a fgspr open set in fuzzy topological space(X, τ). Then λ = fgspr-int(λ). Now λ ∧ fgspr-fr(λ) = fgspr-int(λ) ∧ fgspr-fr(λ) = 0, as by Theorem 6.2. Hence the proof.

(iii) Let λ be a fgspr open set and fgspr closed set in a fuzzy topological space X. This implies that λ = fgspr-int(λ) and λ = fgspr-cl(λ). Then fgspr-fr(λ) = fgspr-cl(λ) – fgspr-int(λ) = 0. Hence the proof.

Theorem 6.4: In a fuzzy topological space (X, τ) the following properties hold for fgspr-fr(λ)

(i) fgspr-int(λ) = λ – fgspr-fr(λ)

(ii) 1 – fgspr-fr(λ) = fgspr-int(λ) ∨ fgspr-int(1 – λ)

Proof:

(i) Since fgspr-fr(λ) = fgspr-cl(λ) ∧ fgspr-cl(1 - λ) by theorem 6.2. Therefore λ – fgspr-fr(λ) = λ – [fgspr-cl(λ) ∧ fgspr-cl(1 – λ)] = [λ – fgspr-cl(λ)] ∨ [λ – fgspr-cl(1 – λ)] = λ – fgspr-cl(1 – λ) = λ ∧ [1 – fgspr-cl(1 – λ)] = λ ∧ [1 – [1 – fgspr-int(λ)] = λ ∧ fgspr-int(λ) = fgspr-int(λ) as by theorem 5.2.

(ii) 1 – fgspr-fr(λ) =1 – [fgspr-cl(λ) ∧ fgspr-cl(1 – λ)] = [1 – fgspr-cl(λ)] ∨ [1 – fgspr-cl(1 – λ)] = fgspr-int(1 – λ) ∨ [1 – [1 – fgspr-int(λ)] = fgspr-int(1 – λ) ∨ fgspr-int(λ) as by theorem 5.2.

VII. Fuzzy semi preregular $T_{1/2}$ space and Fuzzy semi preregular $T^*_{1/2}$ space

Definition 7.1: A fuzzy topological space (X, τ) is called a fuzzy semi preregular $T_{1/2}$ space if every fgspr-closed set is a fuzzy semi-pre closed set.

Theorem 7.2: A fuzzy topological space (X, τ) is a fuzzy semi preregular $T_{1/2}$ space iff every fgspr-open set is a fuzzy semi-pre open set in X.

Proof: Suppose X is a fuzzy semi preregular $T_{1/2}$ space. Let μ be fgspr-open set in X. Then 1–μ is a fgspr-closed set in X. By Definition 7.1, 1– μ is a fuzzy semi-pre closed set in X. Therefore μ is a fuzzy semi-pre open set in X.

Conversely, assume that every fgspr-open set in X is a fuzzy semi-pre open set in X. Let γ be fgspr-closed set in X. Then $1-\gamma$ is a fgspr-open set in X. By hypothesis, $1-\gamma$ is a fuzzy semi-pre open set in X. Therefore γ is a fuzzy semi-pre closed set in X. Hence X is fuzzy semi preregular $T_{1/2}$ space.

Theorem 7.3: Every fuzzy semi preregular $T_{1/2}$ space is fuzzy pre semi $T_{1/2}$ space.

Proof: Let X be a fuzzy semi preregular $T_{1/2}$ space and let γ be fuzzy pre semi closed set in X. By Theorem 3.20, γ is a fgspr-closed set in X and so γ is a fuzzy semi pre closed set in X. Hence X is fuzzy pre semi $T_{1/2}$ space.

Theorem 7.4: Every fuzzy semi preregular $T_{1/2}$ space is fuzzy semi pre $T_{1/2}$ space.

Proof: Let X be a fuzzy semi preregular $T_{1/2}$ space and let γ be fgsp-closed set in X. By Theorem 3.10, γ is a fgspr-closed set in X and so γ is a fuzzy semi pre closed set in X. Hence X is fuzzy semi pre $T_{1/2}$ space.

Theorem 7.5: Every fuzzy semi preregular $T_{1/2}$ space is fuzzy semi pre $T_{1/3}$ space.

Proof: Let X be a fuzzy semi preregular $T_{1/2}$ space and let γ be fgsp-closed set in X. By Theorem 3.10, γ is a fgspr-closed set in X and so γ is a fuzzy semi pre closed set in X. Every fuzzy semi-pre closed set is a fuzzy pre semi closed set. Hence X is fuzzy semi pre $T_{1/3}$ space.

The following example shows that the converse of the above theorems is not true.

Example 7.6: Let X = {a, b, c}, α = {(a, 1), (b, 0), (c, 0)}, β = {(a, 0), (b, 1), (c, 1)}, γ = {(a, 1), (b, 1), (c, 0)} be fuzzy sets of X. Let τ = {0, α, 1}, then β is fuzzy pre semi closed, fgsp-closed and fuzzy semi-pre closed in X. Hence (X, τ) is fuzzy pre semi $T_{1/2}$ space, fuzzy semi pre $T_{1/2}$ space and fuzzy semi pre $T_{1/3}$ space. Now γ is a fgspr-closed set but not a fuzzy semi pre closed set in X. Therefore (X, τ) is not fuzzy semi preregular $T_{1/2}$ space.

Definition 7.7: A fuzzy topological space (X, τ) is called a fuzzy semi preregular $T^*_{1/2}$ space if every fgspr-closed set is a fuzzy closed set.

Theorem 7.8: A fuzzy topological space (X, τ) is a fuzzy semi preregular $T^*_{1/2}$ space iff every fgspr-open set is a fuzzy open set in X.

Proof: Suppose X is a fuzzy semi preregular $T^*_{1/2}$ space. Let μ be fgspr-open set in X. Then 1–μ is a fgspr-closed set in X. By Definition 7.7, 1–μ is a fuzzy closed set in X. Therefore μ is a fuzzy open set in X.

Conversely, assume that every fgspr-open set in X is a fuzzy open set in X. Let γ be fgspr-closed set in X. Then $1-\gamma$ is a fgspr-open set in X. By hypothesis, $1-\gamma$ is a fuzzy open set in X. Therefore γ is a fuzzy closed set in X. Hence X is fuzzy semi preregular $T^*_{1/2}$ space.

Theorem 7.9: Every fuzzy semi preregular $T^*_{1/2}$ space is fuzzy semi preregular $T_{1/2}$ space.

Proof: Let X be a fuzzy semi preregular $T^*_{1/2}$ space and let λ be fgspr-closed set in X. By Definition 7.7, λ is a fuzzy closed set in X. Every fuzzy closed set is fuzzy semi-pre closed set and so λ is fuzzy semi-pre closed set in X. Hence X is fuzzy semi preregular $T_{1/2}$ space.

Theorem 7.10: Every fuzzy semi preregular $T^*_{1/2}$ space is fuzzy $T_{1/2}$ space.

Proof: Let X be a fuzzy semi preregular $T^*_{1/2}$ space and let λ be fg-closed set in X. By Theorem 3.16, λ is a fgspr-closed set in X and so λ fuzzy closed in X by Definition 7.7. Hence X is fuzzy $T_{1/2}$ space.

The following example shows that the converse of the above theorems is not true.

Example 7.11: Let X = {a, b, c}, α = {(a, 0), (b, 1), (c, 0)}, β = {(a, 1), (b, 0), (c, 1)}, γ = {(a, 1), (b, 0), (c, 0)} be fuzzy sets of X. Let τ = {0, α, 1}, then β is fgspr-closed, fg-closed, fuzzy semi-pre closed and fuzzy closed in X. Hence (X, τ) is fuzzy semi preregular $T_{1/2}$ space and fuzzy $T_{1/2}$ space. Now γ is a fgspr-closed set but not a fuzzy closed set in X. Therefore (X, τ) is not fuzzy semi preregular $T^*_{1/2}$ space.

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